MBT-003-1164002 Seat No. ________

M. Sc. (Mathematics) (Sem. IV) (CBCS) Examination

April / May - 2018

Maths : Integration Theory (CMT - 4002)

(New Course)

Faculty Code : 003
Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks

1 Answer the following : **7×2=14**

(a) __________ is a lower semi continues on a topological space X.

(b) Define complete measure on a measurable space and give an example of a complete measure.

(c) True or False? Justify. If γ is a signed measure on $(X, A,)$, $A \in A$ and $\gamma(A) = 0$ then A is the null set w.r.t γ.

(d) The lebesuge measure on $\mathbb{R}$ is __________.

(e) The cumulative function F of a finite barie measure on the real line is __________.

(f) If (X, A, μ) is a complete measure space then {s / s is simple measurable on X and $\mu\{x \in X / s(x) \neq 0\} < \infty\}$ is dense in __________.

(g) Every closed set in a metric set is a __________.

2 Answer any **two** : **2×7=14**

(a) Define signed measure on a measurable space. If μ_1, μ_2 are two signed measures on a measurable spaces (X, A) then state and prove the condition under which $\mu_1 - \mu_2$ is a signed measures on (X, A).

(b) Define positive set w.r.t. a signed measure. Prove that the countable union of positive sets is positive.

(c) State and prove Lebesgue decomposition theorem for a σ–finite measure w.r.t. another σ–finite measure on a measurable space.

3 Answer the following : **2×7=14**

(a) State, without proof, Hahn decomposition theorem. Is Hahn decomposition is unique? Justify.

(b) Prove that if (X, A) is a measurable space and $f : X \to [0, \infty]$ be measurable then there exists a sequence $\{S_n\}_{n=1}^{\infty}$ of simple measurable function such that

(i) $0 \le S_1 \le S_2 \le \cdots .... \le S_n ...... \le f$; on X.

(ii) $\lim_{n \to \infty} S_n = f(x); \forall x \in X$.

OR

(a) State without proof, Jordan decomposition theorem. Is Jordan decomposition is unique? Justify.

(b) Prove that if X be a countable set and μ be the counting measure then $L^P(\mu) \cong l^P; \forall\ 1 \le P < \infty$.

4 Answer any **two** : **2×7=14**

(a) State, without proof, Caratheodary extension theorem. Give an example to show that σ–finite assumption in the theorem cannot be dropped.

(b) State, without proof, Fubim's theorem and Tonelli's theorem.

(c) If μ, γ are measures on a measurable space then with usual notation prove that $\mu << \gamma$ and $\mu \perp \gamma \Rightarrow \mu = 0$. Does $\mu << \gamma \Rightarrow \gamma << \mu$? Justify.

5 Answer any **two** : **2×7=14**

(a) (i) True or False? Justify.

μ is outer regular $\Rightarrow \mu$ is inner regular.

(ii) Define G_δ sets and F_σ sets.

(b) Define :

(i) a locally compact and

(ii) a hausdorff space. Is the set of rationals in $\mathbb{R}$ is locally compact ?

(c) Let X is a locally compact hausdroff space. Prove that Ba(X) = the σ–algebra generated by compact G_δ sets in X.

(d) Define σ–bdd set in a locally compact hausdorff space X. If $E \in Ba(X)$ then prove that either E or X \ E is σ–bdd.